LabVIEW Based Level Control of Coupled Tank System

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Abstract: This paper utilizes Variable Structure Control (VSC) to control the level of a two tank system. By using Sliding Mode Technique in VSC, the levels of the tanks are maintained at a desired setpoint. The system is subjected to the conventional PID controller and the results are brought out. To track the robustness of the system, an external disturbance is intentionally introduced. The ability of disturbance rejection is checked in both VSC and PID controllers. It is found that VSC offers quick settling time, low peak overshoot and low rise time, and as compared to PID control. LabVIEW(Laboratory Virtual Instrument Workbench) is the software used for the control of the whole process. This is very user friendly software. It uses Graphical programming technique. As an extension the control can be implemented by hardware using DAQ cards.

Index Terms: Variable Structure Control, Sliding Mode Technique, PID control, LabVIEW.

I. INTRODUCTION

Variable Structure system contains several subsystems whose parameters can be changed or switched according to the state of the system [1]. The transfer function or the gain of the system can be changed, also the entire system can be changed in such systems. For such variable structure systems, one of the special type of control methods is Sliding Mode Control. Emelynov [2], Utkin[3] proposed and elaborated about Variable Structure systems with Sliding Mode Control (SMC) in their early research studies. This paper presents the level control of a two tank system. Phase canonical form is not employed for implementing SMC. To employ this type of control, the mathematical model of the tanks is derived. The mathematical model obtained is typically a first order system. Sliding Mode Control is applied to the first order system and later the control law is obtained [4]. The results of the controllers are compared and best controller is suggested. LabVIEW is used for simulation. This is used because it is more hardware friendly. The version of the software used is LabVIEW11. LabVIEW is a highly productive development environment that is used by engineers and scientists. This programming is used due to its graphical programming and hardware integration to rapidly design and deploy measurement and control systems [5]. Using LabVIEW the whole process is monitored and controlled. Remote systems are also controlled through internet. This additional feature increases the ease of accessibility of the level process plant parameters even from remote places. The main idea of this paper is to present the response of the system using Variable Structure Control incorporating Sliding Mode Technique and conventional PID control. The results of both the control schemes are compared.

II. SYSTEM DESCRIPTION

This paper is a continuation of modeling of a single tank system [6]. The Fig.1. shows the schematic of a double tank system. It consists of two hold-up tanks which are identical and connected by an orifice. A variable speed pump is used to supply the input which supplies water to the first tank. The orifice present in the bottom of the first tank allows the water to flow into the second tank and in turn out to a reservoir. The objective of this control problem is to adjust the inlet flow rate q(t) so as to maintain the level in the second tank, h2(t) close to a desired set point level, H.
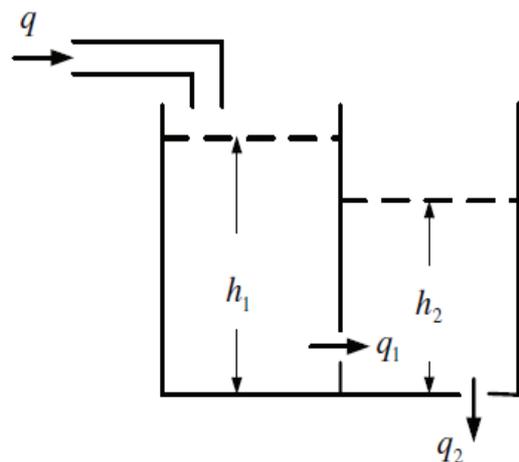

Figure 1. Schematic of the two tank system

The dynamic model of the coupled tanks is represented in (1) and (2).

$$\frac{dh_1}{dt} = \frac{1}{C}(-q_1 + q) \qquad (1)$$

$$\frac{dh_2}{dt} = \frac{1}{C}(q_1 - q_2) \qquad (2)$$

where

$$q_1 = c_{12}\sqrt{2g(h_1 - h_2)} \quad for \ h_1 > h_2$$
$$q_2 = c_2\sqrt{2gh_2} \quad for \ h_2 > 0$$

and

h1(t): Level of first tank;
h2(t): Level of second tank;
q1(t): Flow rate from first Tank to second Tank;
q2(t): Flow rate of second Tank ;
q(t): Inlet flow rate;
g: the gravitational constant;
C: Cross-section area of first Tank and second Tank;
c_{12}: Coupling orifice area;
c_2 : Outlet orifice area.

As for the two tank set up, the fluid flow, q, into Tank 1, cannot be negative because the pump can only pump water into the tank. Therefore, the constraint on the inflow rate is given by (3) and (4).

$$h_1 = -\frac{c_{12}}{c}\sqrt{2g|h_1 - h_2|}sgn(h_1 - h_2) + \frac{1}{c}q \qquad (3)$$

$$h_2 = \frac{c_{12}}{c}\sqrt{2g|h_1 - h_2|}sgn(h_1 - h_2) - \frac{c_2}{c}\sqrt{2gh_2} \qquad (4)$$

At equilibrium, for constant water level set point, the derivatives must be zero, as in (5).

$$h_1 = h_2 = 0 \qquad (5)$$

Thus, at equilibrium, (6) and (7) hold true

$$\frac{c_{12}}{c}\sqrt{2g|h_1 - h_2|}sgn(h_1 - h_2) + \frac{1}{c}q = 0 \qquad (6)$$

$$\frac{c_{12}}{c}\sqrt{2g|h_1 - h_2|}sgn(h_1 - h_2) - \frac{c_2}{c}\sqrt{2gh_2} = 0 \qquad (7)$$

where Q is the equilibrium inflow rate.

As always flow rate is greater than zero and the height of the water is a positive variable. Hence $sgn(h_1 - h_2)$, this implies (8)

$$h_1 \geq h_2 \qquad (8)$$

Let

$$z_1 = h_2 > 0, z_2 = h_1 - h_2 > 0$$

$$z_1 = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \qquad u = q$$

And

$$a_1 = -\frac{c_2}{C}\sqrt{2g}$$
$$a_2 = -\frac{c_{12}}{C}\sqrt{2g}$$

The output of the second tank is the final output of the two tank system. Therefore, the dynamic model can be written in (9) and (10).

$$z_1 = -a_1\sqrt{z_1} + a_2\sqrt{z_2} \qquad (9)$$

$$z_2 = -a_1\sqrt{z_1} - 2a_2\sqrt{z_2} + \frac{1}{C}u \qquad (10)$$

$$y = z_1$$

The objective of the control scheme is to regulate the output $y(t) = z_1(t) = h_2(t)$ to a desired value H. If $y(t) = z_1(t)$ is regulated to a desired value H, then $z_2 = h_1(t) - h_2(t)$ will be regulated to the value $\frac{a_1^2}{a_2^2}H$.

The dynamic model of this system is highly nonlinear. Therefore, a transformation is defined in such a way that the dynamic model is transformed into a form that facilitates the control design.

Let $X = \begin{bmatrix} x \\ x_2 \end{bmatrix}$, and define the transformation x = T(z) as in (11) and (12)

$$x_1 = z_1 \qquad (11)$$

$$x_2 = -a_1\sqrt{z_1} + a_2\sqrt{z_2} \qquad (12)$$

The inverse transformation z = T⁻¹(x) is written as (13) and (14)

$$z_1 = x_1 \qquad (13)$$

$$z_2 = \left(\frac{a_1\sqrt{x_1} + x_2}{a_2}\right)^2 \qquad (14)$$

The dynamic model is written as (15) and (16)

$$x_1 = x_2 \qquad (15)$$

$$x_2 = \frac{a_1 a_2}{2}\left(\left(\sqrt{\frac{z_1}{z_2}} - \frac{z_2}{z_1}\right) + \frac{a_1^2}{2} - a_2^2 + \frac{a_2}{2C}\frac{1}{\sqrt{z_2}}U\right) \qquad (16)$$

Hence, the dynamic model of the system can be written in a compact form as (17) and (18)

$$x_1 = x_2 \qquad (17)$$

$$x_2 = f + \emptyset u \qquad (18)$$
$$y = x_1$$

Where

$$f = \frac{a_1 a_2}{2}\left(\left(\sqrt{\frac{z_1}{z_2}} - \frac{z_2}{z_1}\right) + \frac{a_1^2}{2} - a_2^2\right)$$

$$\emptyset = \frac{a_2}{2C}\frac{1}{\sqrt{z_2}}$$

The dynamic model is used to design control schemes for the two tank system.

III. STATIC SLIDING MODE CONTROLLER FOR A TWO TANK SYSTEM

Let α_s and Ws be positive scalars. Also, let H be the desired constant value of the output of the system.
Define the sliding surface σ_s as (19)

$$\sigma_s = x_1 + \alpha_1(x_1 - H) \qquad (19)$$

$$x_2 = -a_1\sqrt{z_1} + a_2\sqrt{z_2} + \alpha_S(x_1 - H)$$

The sign function is

$$sgn(b) = \begin{cases} +1 & if\ b > 0 \\ 0 & if\ b = 0 \\ -1 & if\ b < 0 \end{cases}$$

$$u = \frac{2C\sqrt{z_2}}{a_2}\left[-\frac{a_1 a_2}{2}\left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right) - \frac{a_1^2}{2} + a_2^2 - \alpha_s\left(-a_1\sqrt{z_1} + a_2\sqrt{z_2}\right)\right]$$

asymptotically stabilizes the output of the system y = z1 = h2 to its desired value H.

The sliding equation mainly depends on (20).

$$\sigma_S = -W_s Sgn(\sigma_s) \qquad (20)$$

The trajectories are associated with the unforced discontinuous dynamics. They reach to zero within a finite time reachability given initial condition provided that the constant Ws is positive. Since σ_s is driven to zero in finite time, the output y = z_1 = h_2 is governed after such a finite time, by the first order dynamics

$$y + \alpha_s(y - H) = 0$$

Therefore, the output y(t) will asymptotically converge to its desired value as α_s is a positive scalar. Hence the static sliding mode controller guarantees the asymptotic convergence of the output y(t) = z1(t) = h2(t) to its desired value H.

The chattering problem affects the proposed control scheme [7]. The chattering is due to the assumption that the control can be switched from one value to another at any moment and with almost zero time delay [8]. The chattering can be reduced by using a boundary layer. Also dynamic sliding mode control can be used to reduce the chattering [9]. To test the effectiveness, a dynamic sliding mode controller is proposed in the next two sections.

IV. DYNAMIC SLIDING MODE CONTROLLER FOR A TWO TANK SYSTEM

To reduce the chattering due to the static sliding mode controller, a dynamic sliding mode controller is proposed in this section [10]. Let α_1, α_2 and Wd be positive scalars. Also, let H be the desired value of the liquid in Tank 2. Define the input-dependent sliding surface σ_d is (21)

$$u = \left(\begin{array}{c} \frac{2C\sqrt{z_2}}{a_2} f_1 + \alpha_1 \left(\frac{a_1 a_2}{2} \left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}} \right) + \frac{a_1^2}{2} - a_2^2 + \frac{a_2}{2C} \frac{1}{\sqrt{z_2}} U \right) + \alpha_2 (-a_1\sqrt{z_1} \pm a_2\sqrt{z_2}) + W_d sgn(\sigma_d) \\ + \frac{1}{2z_2} \left(a_1\sqrt{z_1} - 2a_2\sqrt{z_2} + \frac{1}{c}u \right) u \end{array} \right) \tag{21}$$

With

$$f_1 = \frac{a_1 a_2 (z_1 + z_2)}{4\sqrt{(z_1 z_2)^3}} a_1 z_1{}^{3/2} - 2a_2 z_1 \sqrt{(z_2)} + \frac{1}{c} z_1 u + a_1 z_2 \sqrt{(z_1)} - a_2 z_2{}^{3/2}$$

$$\sigma_d = y_1 + \alpha_1 x_1 + \alpha_2 (x_1 - H)$$

$$\sigma_d = \left(\frac{a_1 a_2}{2} \left(\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}} \right) + \frac{a_1^2}{2} - a_2^2 + \frac{a_2}{2C} \frac{1}{\sqrt{z_2}} U \right) + \alpha_1 (-a_1\sqrt{z_1} \pm a_2\sqrt{z_2}) + \alpha_2 (z_1 - H)$$

asymptotically stabilizes the output of the system y(t) = z1(t) = h2(t) to its desired value H.

Proof: Taking the derivative of sliding surface σ_d. This indirectly indicates (22)

$$\sigma_d = -W_d sgn(\sigma_d) \tag{22}$$

The trajectories associated with the unforced discontinuous dynamics exhibit a finite time reachability to zero from any given initial condition provided that the constant W_d is positive. Since σ_d is driven to zero in finite time, the output y = z1 = h2 is governed after such a finite time, by the second-order dynamics in (23)

$$y + \alpha_1 + \alpha_2 (y - H) = 0 \tag{23}$$

Therefore, the output y(t) will asymptotically converge to its desired value H since α_1 and α_2 are positive scalars. Hence the dynamic sliding mode controller guarantees the asymptotic convergence of the output $y(t) = z_1(t) = h_2(t)$ to its desired value H.

V. RESULTS

The areas of the orifices, c_{12} and c_2, have been experimentally determined using steady-state measurements. The values of these parameters are listed in Table 1. The cross-section area of Tank 1 and Tank 2 are found to be 208.2 cm^2. The gravitational constant is 981 cm/s^2. The desired value of the output of the system is taken to be H = 10 cm.

To obtain realistic results, the simulations are carried out using the following input constraint, 0 cm^3 /s $\leq$ u $\leq$ 50 cm^3/s. Table 1 shows the constants used in the simulation.

TABLE I.
CONSTANTS FOR SIMULATION

Parameters	Values(cm^2)
c_{12}	0.58
c_2	0.24
C	208.2

A.   Static sliding mode controller of double tank system

The controller parameters used in the simulations are taken to be α_s = 0.1 and Ws = 10 after several iterations. The following figures show the simulation results when the static sliding mode controller is used. It can be seen from Fig. 2. that the output y(t) = z_1(t) = h_2(t) converges to its desired value H in about 160 s. The control input u(t) = q(t) is shown in Fig. 3. chattering is evident in this graph.
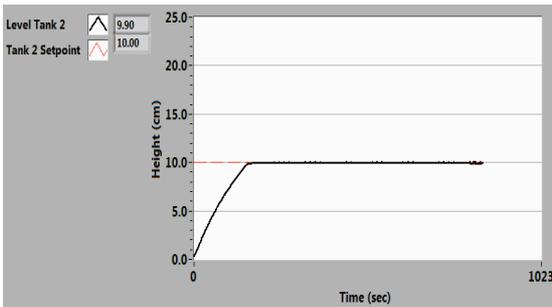

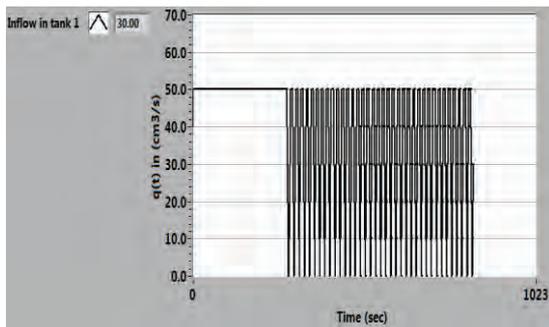Figure 2. Liquid tank level in Tank 2 when static sliding mode controller is used.

Figure 3. Inflow rate into Tank 1 when static sliding mode controller is used.

B.   Dynamic sliding mode controller of a two tank system

The controller parameters used in the simulations are taken to be α_1 = 1, α_2 = 2, and W_d = 5. The following figures show the simulation results when the dynamic sliding mode controller is used. It can be seen from Fig. 4. that the output y(t) = z_1(t) = h_2(t) converges to its desired value H in about 220s. The control input u(t) = q(t) is shown in Fig. 5. note that chattering [12] is greatly reduced.
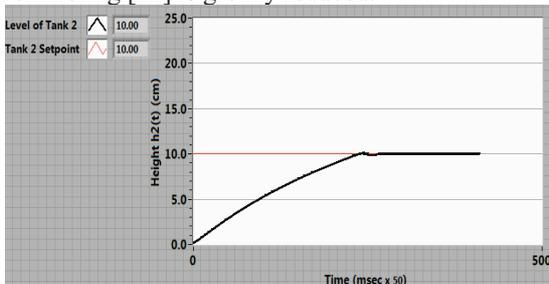

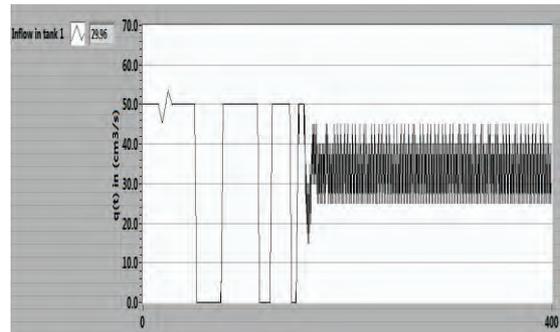Figure 4. Level of Liquid in Tank 2on application of Dynamic sliding mode controller.

Figure 5. The input flow rate of tank 1 on application of Dynamic Sliding Mode controller.

C. PID controller of a two tank system

The controller parameters used in the simulations are taken to be Kc = 5.8, Ti = 2.4, and Td = 0.5. The tuning used is zeigles nicolos method [11] It can be seen from Fig. 6. that the output y(t) = z_1(t) = h(t) converges to its desired value H (setpoint) in about 135 s. The control input u(t) = q(t) is shown in Fig. 7. note that there is an overshoot.
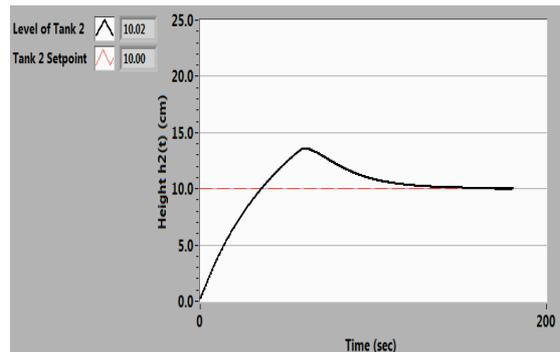

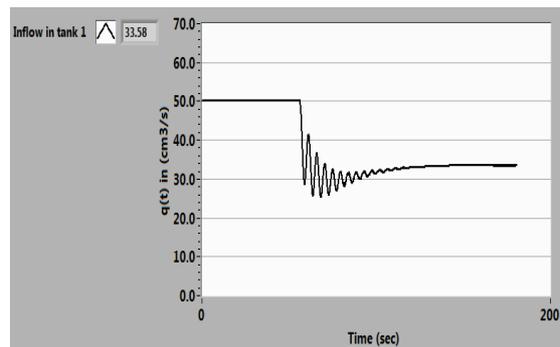Figure 6. Liquid tank level in Tank 2 when PID controller is used.

Figure 7. Inflow rate into Tank 1 when PID controller is used.

VI. CONCLUSIONS

The following are the conclusions derived from the above simulations. It is found that PID controller had an overshoot whereas both the SMC and Dynamic SMC didn't exhibit peak overshoot. The settling time were 135s, 220s and 160s for PID, Dynamic SMC and SMC respectively indicating PID controllers were faster than the other two controllers. But the chattering in the flow rate response is minimal in the Dynamic SMC [12] thereby making this controller more suitable for maintaining less wear and tear especially for the final control elements.

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