Ductility of Reinforced Concrete Beams

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Abstract---**Concrete structures are designed by satisfying two criteria namely, safety and serviceability. Safety criterion checks whether the structure is safe in carrying the designed load or not. Serviceability criterion checks whether the developed deflections and crack widths are within the permissible limits or not. The evaluation of adequate margin of safety of concrete structures against failure can be assured by the accurate prediction of ultimate load and the complete Moment-curvature response. If the moment-curvature relationship is available, then the strength, stiffness and the ductility characteristics of the cross-section can be predicted. In this paper, an attempt has been made to study the variation of the strength, stiffness and the ductility of the singly reinforced concrete beams of M25 and M35 grades, using Moment-Curvature diagrams. Working Stress Method and Limit State Method are used for designing the reinforced concrete beams. Force equilibrium equations, Strain compatibility equations and Material models are used in developing the Moment-curvature diagrams. The Concrete strain and the percentage of tension steel are the input parameters. It is observed that for a given percentage of Tension Steel, the Moment and the Curvature are increasing with respect to the increase in the Concrete strain. But the decrease in Curvature value with the increase in percentage of Tension Steel reduced the energy absorption capacity (ductility).**

Index Terms---**Force Equilibrium, Strain Compatibility, Moment-Curvature, Strength, Stiffness, Ductility.**

I. Introduction

Curvature is defined as the rotation of the member per unit length. In a reinforced concrete beam under bending, the moment-curvature relationship is assumed to fully represent the structural response of the beam cross section [1]. The moment-curvature relationship for reinforced concrete sections can be generated using the Equilibrium equations, the Compatibility equations and the Material models [2].

Ductility is defined as the ability of the member to undergo inelastic deformation beyond initial yield deformation with no increase in the load resistance. Ductility depends on the (i) Strength and (ii) Stiffness of the member. Strength is the ability of the member to resist the loads [3]. Stiffness is the ability of the member to resist the deformation. Ductility is defined with respect to the strains (Strain ductility), curvatures (Curvature ductility) and deflections (Deflection ductility) [4]. Strain ductility depends on the type of material. It is defined as the ratio of ultimate strain to yield strain. Strain ductility is studied using stress-strain diagrams. Curvature ductility depends on the size and the shape of the cross section of the member. Curvature ductility is defined as the ratio of curvature at ultimate load to curvature at yield load. Curvature ductility is studied using Moment-curvature diagrams. Deflection ductility depends on the entire configuration of the member and the entire loading acting on the member. Deflection ductility is defined as the ratio of ultimate deflection to yield deflection. Deflection ductility can be studied using load-deflection diagrams [5][6][7].

II. Significance Of Ductility

Ductility is very important criterion in the design of structures in earthquake prone regions. Increase in the ductility of the structure reduces the induced seismic forces and hence reduces the probability of the collapse of the structure. The possible distribution of bending moment, shear force and axial load used in the design of statically indeterminate structures depend on the ductility of the members at the critical sections. Moment-curvature relationship predicts the strength, stiffness and the ductility characteristics of the cross-section. The evaluation of adequate margin of safety of concrete structures against failure can be assured by the accurate prediction of the ultimate load and the complete moment-curvature response.

III. Algorithm For Moment- Curvature Diagram In Working Stress Method

A. *Assumptions*

Assumptions made in the Working Stress Method are 1. Plane sections before bending remain plane after bending. 2. Steel resists all the tensile stresses. 3. Stress distribution across the concrete section is linear. 4. Material model for the steel is elastic-plastic. 5. Bond between the steel and the concrete is perfect. 6. Maximum strain in the concrete under bending compression is 0.0035. 7. Modular ratio (m) is assumed as $m = (280/3\sigma_{cbc})$.

The variation of stress and strain across the beam section is presented in Fig.1.

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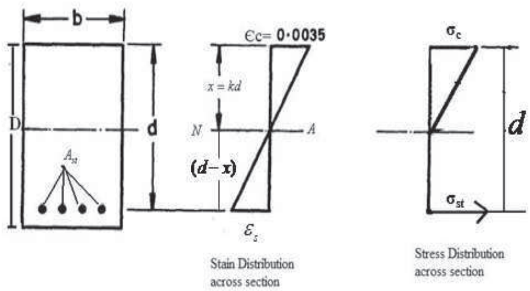

Fig. 1 Variation of Stress and Strain across the beam section

B. Material Model For The Concrete In Compression

The material model for the Concrete in compression is assumed as shown in Fig.2.

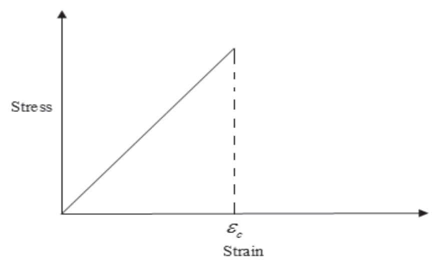

Fig. 2 Material Model for the Concrete in Compression

C. Material Model for the Steel in Tension

The material model for the Steel in tension is taken as presented in Fig.3.

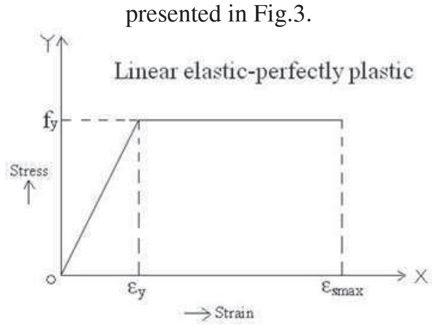

Fig. 3 Material Model for the Steel in Tension

D. Algorithm

Step-wise procedure for developing the Moment-curvature relationship of singly reinforced Concrete beams is detailed below:

(i) Compressive strength of the concrete (f_{ck}) is assumed.
(Range: f_{ck}=20MPa to 50MPa)

(ii) Concrete modulus (E_c) is determined using Equation (1)

$$E_c = 5000\sqrt{f_{ck}} \tag{1}$$

(iii) Concrete strain (ϵ_c) is assumed from 0.0001 to 0.0035.

(iv) Concrete stress (σ_c) is determined using the Equation (2)

$$\sigma_c = E_c \times \epsilon_c \tag{2}$$

(v) Steel modulus of elasticity (Es) is assumed as Es=2x10□ Mpa.

(vi) Percentage of tension steel (ρ) is assumed from 0.2% to 1.4%.

(vii) Neutral axis depth (χ) is calculated using the Force Equilibrium Equation (3),
Total Compressive force (C_u) = Total Tensile force(T_u)

$$\tfrac{1}{2}\sigma_c bx = \sigma_{st} A_{st} \tag{3}$$

(viii) Yield strain (ϵ_y) of steel is calculated using Equation (4)

$$\varepsilon_y = \frac{\sigma_y}{E_s} \tag{4}$$

(ix) If the strain in the steel (ϵ_s) is less than the yield strain, ϵ_y, then ϵ_s is calculated using the Equation(5)
and used in the neutral axis depth calculation.

$$\varepsilon_s = \left(\frac{d-x}{x}\right)\varepsilon_c \tag{5}$$

(x) If $\epsilon_s > \epsilon_y$, then the strain in the steel (ϵ_s) is calculated using the Equation (6)and used in the calculation of modified neutral axis depth..

$$\varepsilon_s = \left(\frac{f_y}{E_s}\right) \tag{6}$$

The modified neutral axis depth is calculated using the Equation (7).

$$\frac{x}{d} = \frac{\rho f_y}{0.5\sigma_c} \tag{7}$$

(xi) Moment of resistance (M) of the beam section is calculated using the Equation (8).

$$M = \frac{1}{2}\sigma_c \frac{x}{d}\left[1 - \left(\frac{1}{3}\left(\frac{x}{d}\right)\right)\right] \tag{8}$$

(xii) Curvature (φd) of the beam section is calculated using the Equation (9)

$$\phi d = \frac{\varepsilon_c}{(\frac{x}{d})} \tag{9}$$

Moment-curvature diagrams for M25 and M35 grades of singly reinforced concrete beam are developed using the force equilibrium equations, strain compatibility equations and the material models. The concrete strain and the percentage of tension steel are the input parameters. The variation of Moment and Curvature is presented in Fig. 4, Fig. 5 and Fig. 6 respectively.

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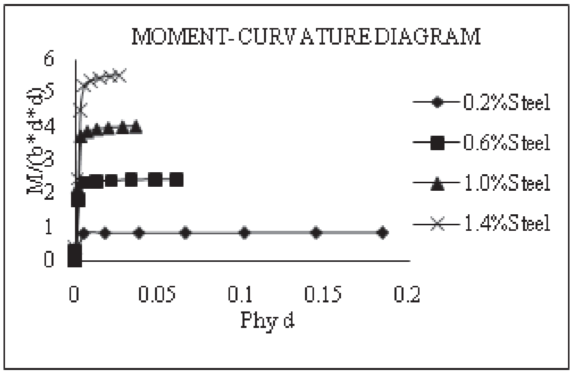

Fig. 4 Moment-Curvature diagram for M25 Grade R.C.Bean (Working Stress Method)

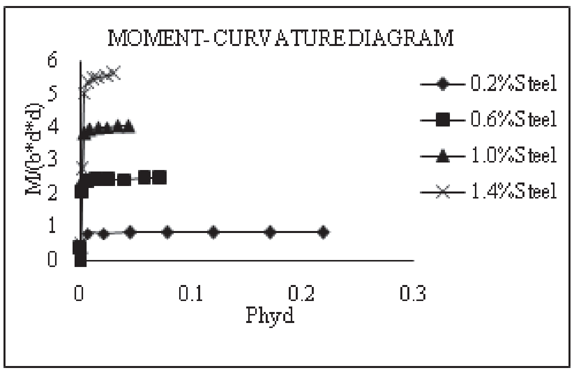

Fig. 5 Moment-Curvature diagram for M35 Grade R.C.Bean (Working Stress Method)

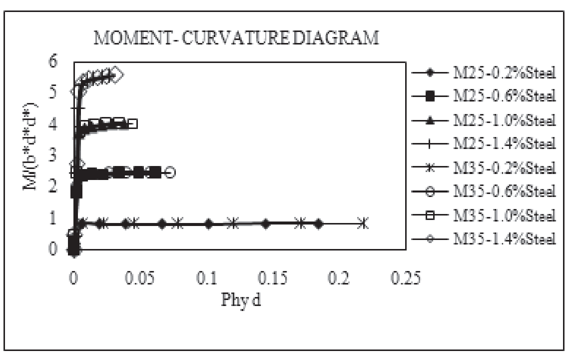

Fig. 6 Moment-Curvature diagram for M25 and M35 M35 Grade R.C.Bean (Working Stress Method)

IV. Algorithm For Moment- Curvature Diagram In Limit State Method

A. Assumptions

Assumptions made in the Limit State Method are

1. Plane sections before bending remain plane after bending. 2. Steel resists all the tensile stresses. 3. Material model for the steel is Elastic-plastic. 4. Material model for the concrete is rectangular parabola. 5. Bond between the concrete and the steel is perfect. 6. Maximum strain in the concrete under bending compression is 0.0035. 7. Maximum strain in the tension reinforcement at failure shall not be less than [$(0.85/f_y) + (0.002)$].

The variation of stress and strain across the beam section is presented in Fig. 7.

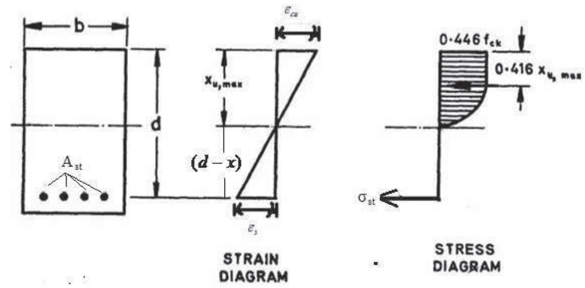

Fig. 7 Variation of Stress and Strain across the beam section

B. Material Model for the Concrete in Compression

The material model for the Concrete in compression is assumed as shown in Fig. 8.

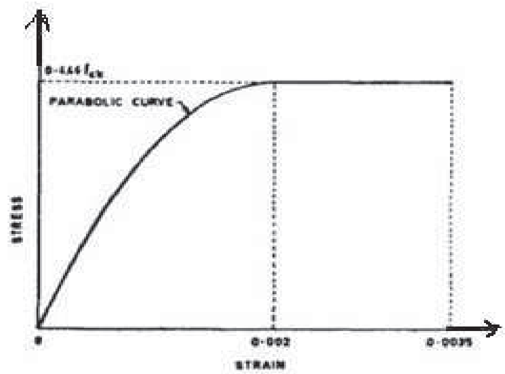

Fig. 8 Material model for the Concrete in Compression

C. Material Model for the Steel in Tension

The material model for the Steel in tension is taken as shown in Fig. 9.

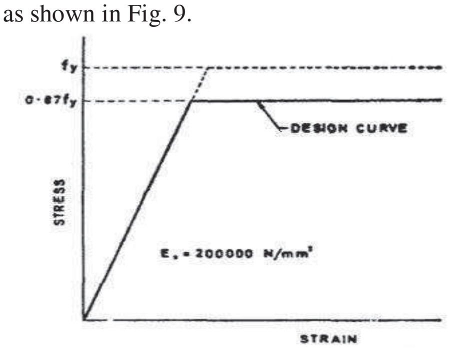

Fig. 9 Material model for the Steel in Tension

D. Algorithm

Step-wise procedure for developing the Moment-curvature relationship of singly reinforced Concrete beams is detailed below:

(i) Compressive strength of the concrete (f_{ck}) is assumed. (Range: f_{ck}=20MPa to 50MPa)

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(ii) Concrete modulus (E_c) is determined using the Equation (10).

$$E_c = 5000\sqrt{f_{ck}} \tag{10}$$

(iii) Concrete strain (ϵ_c) is assumed from 0.0001 to 0.0035.

(iv) Concrete stress (σ_c) is determined using the Equation (11).

$$\sigma_c = E_c \times \epsilon_c \tag{11}$$

(v) Steel modulus of elasticity (Es) is assumed as Es=2x10□ Mpa.

(vi) Percentage of tension steel (ρ) is assumed from 0.2% to 1.4%.

(vii) Neutral axis depth (χ) is calculated using the Force Equilibrium Equation (12),

Total Compressive force (C_u) = Total Tensile force (T_u)

$$\left[\frac{2}{3}(b)(0.57x_u)(0.446f_{ck})+(b)(0.43x_u)(0.446f_{ck})\right]=\left[(E_s)\left(\frac{d-x}{x}\right)(\varepsilon_c)\left(\frac{\rho bd}{100}\right)\right] \tag{12}$$

(viii) Yield strain (ϵ_y) of steel is calculated using the Equation (13) or (14).

For Mild Steel

$$\varepsilon_y = \frac{0.87 f_y}{E_s} \tag{13}$$

For HYSD steel

$$\varepsilon_y = \frac{0.87 f_y}{E_s} + 0.002 \tag{14}$$

(ix) If the strain in the steel (ϵ_s) is less than the yield strain, ϵ_y, then ϵ_s is calculated using the Equation (15) and used in the neutral axis depth calculation.

$$\varepsilon_s = \left(\frac{d-x}{x}\right)\varepsilon_c \tag{15}$$

(x) If $\epsilon_s > \epsilon_y$, then ϵ_s is taken as equal to ϵ_y.

ϵ_y is calculated using the Equation (16).

$$\varepsilon_y = E_s\left(\frac{0.87 f_y}{E_s}\right)\left(\frac{\rho bd}{100}\right) \tag{16}$$

If $\epsilon_s > \epsilon_y$, then the modified neutral axis depth is calculated using the Equation (17).

$$\frac{x}{d} = \frac{0.87 f_y \rho}{0.36 f_{ck}} \tag{17}$$

(xi): Moment of resistance (M) of the beam section is calculated using the Equation (18).

$$\frac{M}{bd^2} = 0.36\left(\frac{x}{d}\right) f_{ck}\left[1-0.42\left(\frac{x}{d}\right)\right] \tag{18}$$

(xii): Curvature (φd) of the beam section is calculated using the Equation (19).

$$\phi d = \frac{\varepsilon_c}{\left(\frac{x}{d}\right)} \tag{19}$$

Moment-curvature diagrams for singly reinforced concrete beam of M25 and M35 grades are developed using the force equilibrium equations, strain compatibility equations and the material models. The concrete strain and the percentage of tension steel are the input parameters. The variation of Moment and Curvature is presented in Fig. 10, Fig. 11 and Fig. 12 respectively.

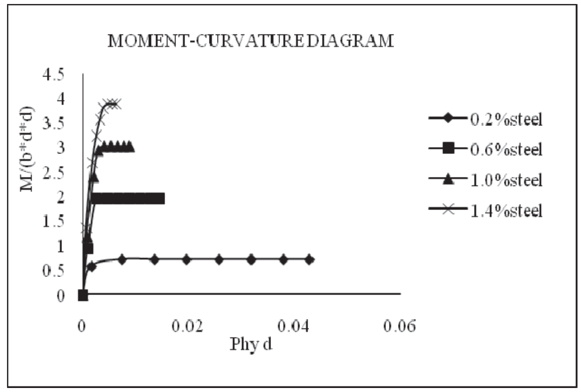

Fig. 10 Moment-Curvature diagram for M25 Grade R.C.Beam (Limit State Method)

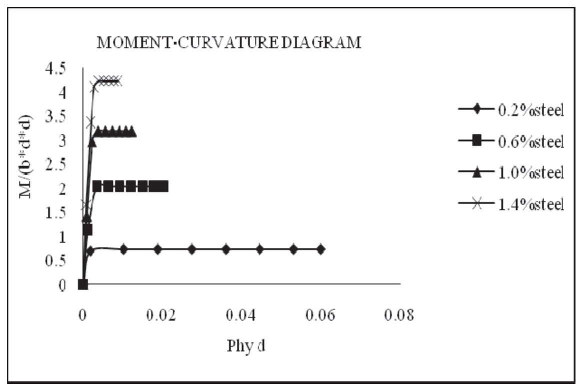

Fig. 11 Moment-Curvature diagram for M35 Grade R.C.Beam (Limit State Method)

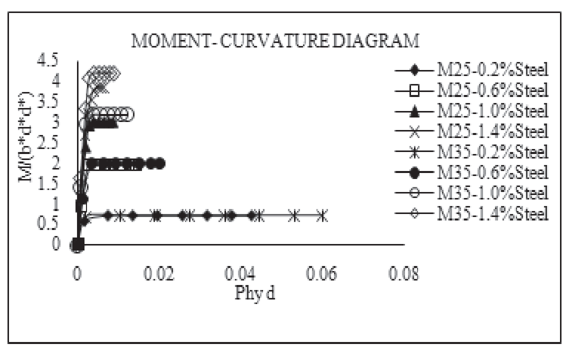

Fig. 12 Moment-Curvature diagram for M25 and M35 Grade R.C.Beam (Limit State Method)

IV. Discussions And Conclusions

Moment-Curvature diagrams are developed for singly reinforced concrete beams in Working Stress method and Limit State method. The beams are made with the M25 grade concrete and the M35 grade concrete. Programming

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is done in Spreadsheets for the development of Moment-curvature diagrams. Input parameters used in the programming are Grade of Concrete (f_{ck}), Grade of Steel (f_y), Concrete strain (ϵ_c) and percentage of Tension steel (ρ). According to the Indian Standard Code of Practice for Plain and Reinforced Concrete Structures [8], the minimum percentage of tension steel (ρ) is taken as 0.2%. The maximum percentage of tension steel used is 1.4%. The percentage increment of tension steel is 0.4%.

A. Moment-Curvature Diagrams- Working Stress Method and Limit State Method

M25 Grade Concrete

Keeping f_{ck} constant, Moment-Curvature diagrams are developed by varying the concrete strain from 0.0001 to 0.0035. For a particular percentage of tension steel, the curvature is found to be increasing with the increase in the concrete strain. But the curvature is observed as decreasing with the increase in the percentage of Tension Steel. Moment resisting capacity is found to be increasing in this case. The decrease in the curvature with the increase in the percentage of Tension Steel presented in Fig. 4, Fig. 5, Fig. 10 and Fig. 11 indicates that the energy absorption capacity (Ductility) of reinforced concrete beams is reduced. It is because the concrete first reaches its permissible stress before the steel reaches its permissible stress thereby giving brittle failure.

M35 Grade Concrete

Moment-Curvature diagrams are developed for the singly reinforced concrete beams of M35 grade and the similar trend is observed for the Moment resisting capacity and the Curvature as observed in the M25 grade singly reinforced concrete beams.

Comparison of Moment-Curvature variation for M25 Grade and M35 Grade Singly Reinforced Concrete Beams

Keeping percentage of the tension steel constant and varying the concrete grade from M25 to M35, the moment resisting capacity and the curvature are found to be increasing as presented in Fig. 6 and Fig. 12. This variation is observed when the concrete strain is increased from 0.0001 to 0.0035. It is because when the strain in the concrete increases, stress in the concrete also increases. Increase in the concrete stress decreases the neutral axis depth factor (x/d), thereby increasing the curvature of the member.

B. Comparison of Moment-Curvature variation-Working Stress Method and Limit State Method

M25 Grade and M35 Grade Singly Reinforced Concrete Beams

For a particular percentage of Tension Steel, when the design method is changed from the Limit State Method to the Working Stress Method, Moment and Curvature are found to be decreasing with the increase in the concrete strain from 0.0001 to 0.0035. Similar trend is observed when the percentage of Tension steel is increased from 0.2% to 1.4% in increments of 0.4%. This is because of the smaller partial safety factors of the concrete and the steel in the Limit State Method compared to the factors of safety of the concrete and the steel in the Working Stress Method. Due to this difference in safety factors of the concrete and the steel, the materials utilization capacities will be more in the Limit State Method for the concrete and the steel. Therefore, the members designed based on Limit State design are slender when compared to the members designed based on the Working Stress Method. Due to the slenderness of the members in the Limit State Method, the moment carrying capacity and the curvature are smaller compared to the moment carrying capacity and the curvature in the Working Stress Method. The Moment resisting capacities and the curvatures are presented in the Table.1 and the Table.2 respectively. Based on the curvature at the first yield and the curvature at the ultimate, the ductility index is calculated and presented in the Table.1 and the Table.2 respectively. Ductility Index is the ratio of the curvature at the first yield point to the curvature at the ultimate point. More the value of the ductility index, more will be the energy absorption capacity for the member. Member with higher ductility index will have gradual failure giving warning signs through fine cracks with more curvature before its complete collapse. Same trend has been observed for M35 grade reinforced concrete beams.

TABLE. 1

CURVATURE AT FIRST YIELD, CURVATURE AT ULTIMATE AND DUCTILITY INDEX VALUES

Working Stress Method						
	M25 Grade concrete			M35 Grade concrete		
Percentage of Tension Steel	Curvature at Yield (°/mm)	Curvature at Ultimate (°/mm)	Ductility Index	Curvature at Yield (°/mm)	Curvature at Ultimate (°/mm)	Ductility Index
0.2	0.01822	0.18449	10.12569	0.02156	0.21829	10.12477
0.6	0.00607	0.06149	10.13015	0.00719	0.07276	10.11961
1.0	0.00364	0.03689	10.13462	0.00432	0.04366	10.10648
1.4	0.0029	0.02636	9.08966	0.00314	0.0281	8.94904

TABLE. 2

CURVATURE AT FIRST YIELD, CURVATURE AT ULTIMATE AND DUCTILITY INDEX VALUES

Limit State Method						
	M25 Grade concrete			M35 Grade concrete		
Percentage of Tension Steel	Curvature at Yield (°/mm)	Curvature at Ultimate (°/mm)	Ductility Index	Curvature at Yield (°/mm)	Curvature at Ultimate (°/mm)	Ductility Index
0.2	0.00732	0.04271	5.83470	0.01025	0.0598	5.83415
0.6	0.00248	0.01447	5.83468	0.00347	0.02022	5.82709
1.0	0.00159	0.00871	5.47799	0.00226	0.01219	5.39381
1.4	0.00172	0.00623	3.62209	0.00284	0.00872	3.07042

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