Moment Capacity of Reinforced Concrete Beam Including Uncertainties

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Abstract – **The purpose of this paper is to evaluate the reliability of reinforced concrete beam designed under the provisions of IS: 456- 2000 including the numerous sources of uncertainties in the load and resistance-related parameters, which are usually not included in the theoretical method of calculating the Ultimate Moment Capacity of a reinforced beam. Based upon the statistical results, the possible random variables are obtained from large experimental data. In this attempt, the probability of failure of a reinforced concrete beam is obtained by calculating Ultimate Moment Resisting Capacity. These random variable distribution curves, incorporating various uncertainties are included, excluding the partial safety factors, using a high level computing language i.e., MATLAB is used for our work.**

Keywords: **Reliability, Ultimate moment carrying capacity, Reinforced Concrete Beam, Partial Safety Factors.**

I. Introduction

In this theoretical method of calculating the ultimate moment carrying capacity of a reinforced concrete (RC) beam, we include partial safety factors for materials and neglect the various uncertainties included during the construction like variation in material properties, geometric dimensions and other load and resistance parameters which are random in nature.

Various studies have been conducted globally in order to obtain the probabilistic approach of variations in the design parameters. Based upon the large experimental data available the nature, distribution of random variables is attained. Using these possible random variables, a large mathematical and experimental data is generated using the various simulation techniques available and the reliability of a given structure is calculated for an array of inputs given.

II. Research Methodology

Ultimate Moment of Resistance (Mur) of an RC beam without considering the partial safety factor:

The code has adopted the usage of partial safety factors as given below:

$$R_d \geq S_d$$

R_d **is** the design resistance is computed using reduced material strengths, involving two separate partial safety factors γ_c (for concrete) and γ_s (for steel). S_d is the design load effect computed for enhanced loads involving separate partial safety factors. It may be noted that the partial safety factors of materials (γ_c, γ_s) are greater than unity and are the dividing factors while the multiplication factor ϕ is less than unity leading to overestimation of loads and underestimation of material strength resulting in improved safety. But in this case the partial safety factors are not taken into consideration. The nominal strength of concrete 0.67fck and nominal yield strength of steel (fy) on the side of resistance and nominal load effects are taken as such providing nominal results.

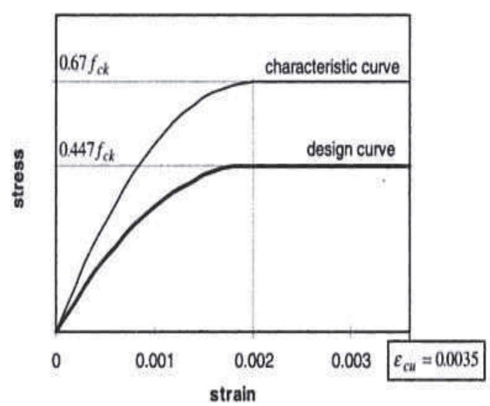

Figure: 1 Characteristic and design stress- strain curves of concrete

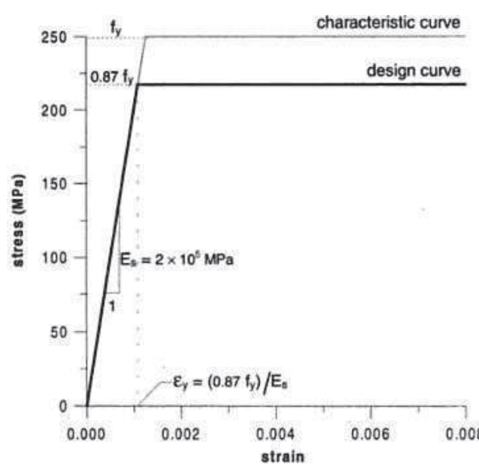

Figure: 2 Characteristic and design stress- strain curves for Fe 250 grade cold worked steel.

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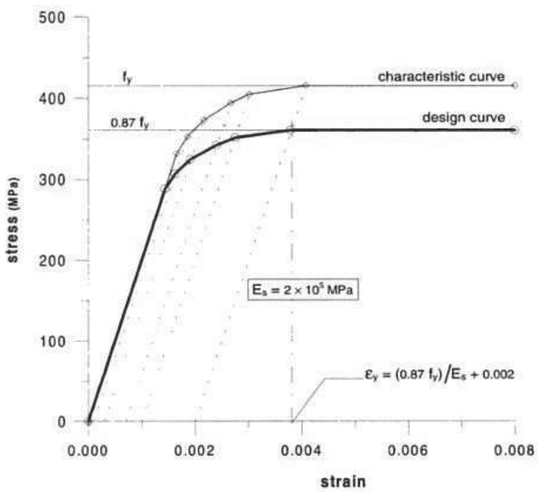
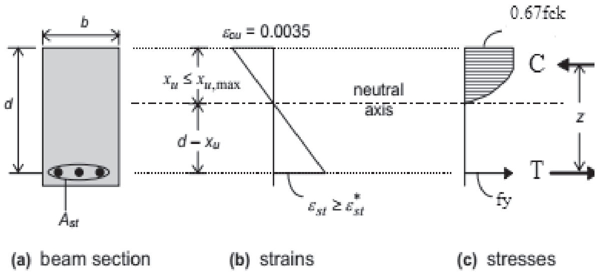


Figure: 3 Characteristic and design stress- strain curves for Fe 415 grade cold worked steel.

The characteristic curve is taken into consideration rather than the usual design curve, for computing the ultimate moment of resistance of a beam.



Steel Grade	Fe 250	Fe 415	Fe 500
$X_{u,\,max}/d$	0.518	0.462	0.4375

A) Analysis of Singly Reinforced Rectangular Sections

Analysis of a given reinforced concrete section at ultimate limit state of flexure implies the determination of the ultimate moment of resistance $\mathbf{M_{ur}}$ of the section, which is given by the couple resulting from flexural stresses:

Figure: 4 Behaviour of singly reinforced rectangular section in flexure

$$M_{ur} = C.Z = T.Z$$

Z= Lever arm distance

$$T = f_{st}.A_{st}$$

$f_{st} = f_y$, for $X_u \le X_{u,max}$ and the line of action of T corresponds to the level of the centroid of the tension steel.

In order to compute the magnitude of the resultant compressive forces, the concrete stress block in compression must be analysed such that nominal strength of concrete 0.67fck is taken into calculations rather than design strength of concrete 0.447fck. Therefore the resulting compressive force for a given beam with width 'b', effective depth 'd' and neutral axis at a depth of X_u from the topmost compression fibre is given as:

$$C = 0.542.fck.b.Xu$$

The line of action of 'C' is the centroid of the stress block located at a distance X' from the extreme fibre of concrete experiencing the maximum compressive strain.

$$X' = 0.416\ X_u$$

(a) Depth of Neutral Axis:

The depth of neutral axis is such that where C = T, satisfying the equilibrium of forces.

$$X_u = \frac{f_{st} * A_{st}}{0.542 * f_{ck} * b}$$

When $X_u \le X_{u,\,max}$, $f_{st} = f_y$,

When $X_u > X_{u,\,max}$, $\varepsilon_{st} < \varepsilon^*{st}$, implies that steel may not yield at the ultimate limit state in flexure. Therefore $f_{st} < f_y$, the true location of neutral axis is obtained by trial and error method, also called as strain compatibility method,

(b) Ultimate Moment of Resistance Without Considering Partial Safety Factors

The lever arm for a singly reinforced rectangular section is given by:

$$Z = d - 0.416X_u$$

$$M_{ur} = 0.542.\ f_{ck}*b*\ X_u\ *(d - 0.416\ X_u)$$

$$M_{ur} = f_{st}.A_{st}\ (d - 0.416\ X_u)$$

(c) Limiting Depth of Neutral Axis

Table 1. Limiting depth of neutral axis without considering partial safety factors

(d) Percentage of steel

Percentage of tension steel is given by:

$$P_t = \frac{A_{st} * 1000}{b * d} = \frac{0.542 f_{ck} * X_u * 100}{f_y * d}$$

When $X_u = X_{u,\,max}$

$$P_{t,lim} = \frac{0.542 f_{ck} * X_{u,max.} * 100}{f_y * d}$$

B) Analysis of Doubly Reinforced Rectangular Sections

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In a doubly reinforced section the condition of force equilibrium applied is:

$$C_c + C_s = T$$

Where:

$C_c = 0.542 * f_{ck} * b * X_u$ (Resultant compressive force in concrete)

$$C_s = (f_{sc} - 0.67 f_{ck}).A_{sc}$$

T = Resultant tension force in steel = $f_{st}.A_{st}$

(a) Depth of neutral axis

Case (i) Pt > Pc

Considering the force equilibrium condition and without considering partial safety factors the neutral axis depth for a doubly reinforced beam in which percentage tension steel (Pt) is greater than the percentage compression steel (Pc) can be calculated as that of a singly reinforced beam. Assuming both the tension and compression steel yielded and hence fst = fy = fsc, the neutral axis depth is given by

$$X_u = \frac{f_{st}A_{st} - (f_{sc} - 0.67 f_{ck})\, A_{sc}}{0.542.\, f_{ck}.b}$$

The conditions are applicable as discussed earlier in singly reinforced beam for & f_{sc}, when necessary strain compatibility method is to be employed.

Case (ii) Pt < Pc

When percentage of tension steel is much less than percentage of compression steel leading to an over compression reinforced beam section, the tension resistance of the beam becomes very less. When the P_c is much greater than P_t, there exists a tendency of the neutral axis depth to move much closer towards the compression steel and at times causes the reversal of stresses in the compression steel.

Hence, the above furnished formula for calculating the X_u cannot be used directly. Hence trial and error approach for finding the depth of neutral axis can be applied. For a given beam the force equilibrium conditioned is verified by taking a continuous series of values for X_u with any required increment starting from zero. Corresponding strains and stresses are calculated and the value of X_u, where the condition of force equilibrium is satisfied is taken as the true location of neutral axis depth. This method is very flexible and is applicable for any type of beam given but when done manually, it is very tedious.

(B) Ultimate Moment of Resistance Without Considering Partial Safety Factors

The moments of C_c and C_s about the centroid of tension steel give the ultimate moment of resistance:

M_{ur}= 0.542. f_{ck}.b. X_u (d – 0.416 X_u) + (f_{sc} - 0.67 f_{ck}) A_{sc} (d-d')

Where: d'= Distance between the centroid of the compression steel and the extreme fibre in the concrete.

f_{sc} = Stress in the compression steel

Other parameters remain the same as discussed earlier.

III. Experimental Procedure

Algorithm for finding the Ultimate Moment of Resistance of RC beams without considering partial safety factors:

The furnished algorithm given below is applicable for both singly reinforced and doubly reinforced beam (including case (i) & case (ii)). Based on this algorithm, a MATLAB code is generated to find the ultimate moment of resistance of any beam for any percentage of tension and compression steel provided, without considering the factor of safety of materials.

In the following algorithm, the true neutral axis depth is obtained by trial and error approach by equating the force equilibrium condition for values from 0.001 to 1 for the ratio of neutral axis to effective depth of beam and the program terminates when the condition is satisfied. Then the ultimate moment of resistance is calculated corresponding to the true neutral axis depth.

1) Given beam of width (b), effective depth (d), distance between the centroid of compression steel and the topmost compression fibre (d'), percentage of tension steel (P_t), percentage of compression steel (P_c), grade of steel (f_y) and grade of concrete(f_{ck})

2) For X_u /d value starting from 0.001 to 1, strains in tension and compression steel are calculated using respective formulae.

3) Using the strain values obtained the stresses in the tension and compression steel are calculated from the respective nominal stress strain curve of the steel (as per grade of steel provided). For Fe 415 & Fe 500, co-ordinate points for nominal stress and their respective strains in the parabolic path of nominal stress-strain curve, as shown in the figure 5, are given below in table 2.

4) Assuming the parabolic path to be continuous fragmented straight line, stress values are calculated using piece-wise linear interpolation. The stress points in the curve chosen are given, along with their respective proof strain values, in table 2

Table 2

Salient Points on The Stress-Strain Curve for Cold-Worked Steel

Nominal stress	$0.8f_y$	$0.85f_y$	$0.9f_y$	$0.95f_y$	$0.975f_y$	f_y
Proof strain	0	0.0001	0.0003	0.0007	0.001	0.002

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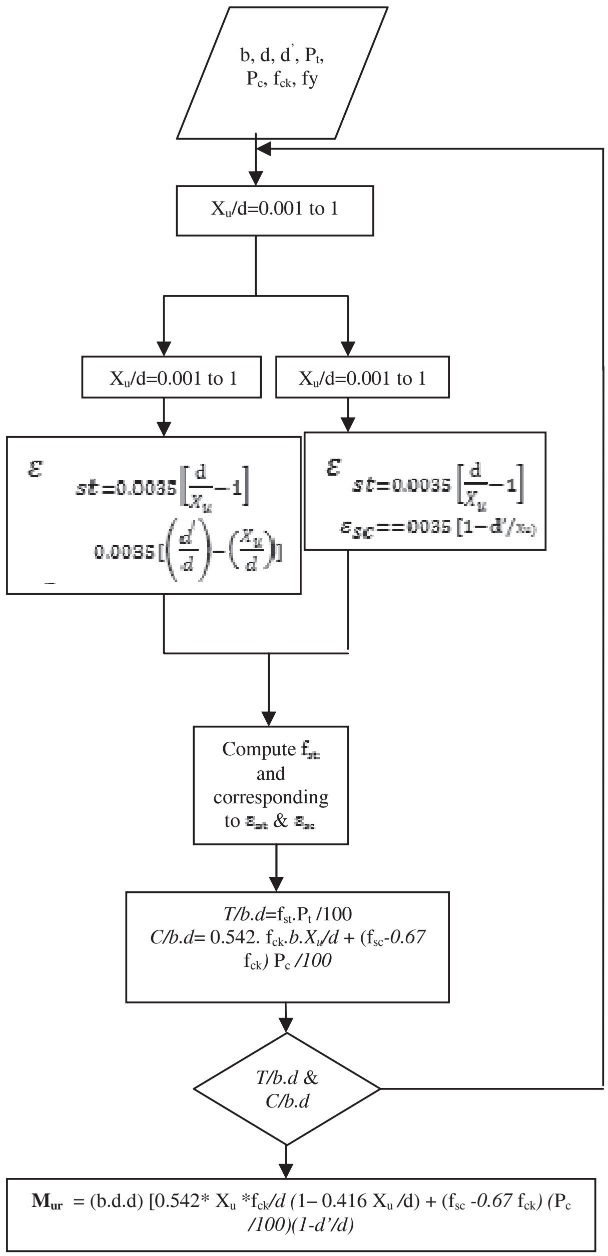

Figure 5 Algorithm for calculating the moment carrying capacity of a Reinforced Concrete beam.

TABLE 3

CHARACTERISTIC STRESS-STRAIN VALUES

Fe 415		Fe 500	
Stress	Strain	Stress	Strain
0.00166	332.005	0.002	399.97
0.00186	352.705	0.002225	425.385
0.002167	373.52	0.00255	449.995
0.00267	394.22	0.003075	474.95
0.00302	404.57	0.0034375	487.485
0.004075	415.036	0.0045	500.02

5) The tension and compression forces for the corresponding stresses are calculated and force equilibrium condition is checked. If not satisfied, iterations continue for the next value for X_u /d. If force equilibrium is satisfied, then calculate the ultimate moment of resistance of the beam using the formula for M_{ur} aforementioned.

Note: While considering the ratio of neutral axis to the effective depth of the beam, one must be very cautious about the strain compatibility equation for computing the strains in compression steel. The reason is that there will be a change in similar triangles and accordingly their formulae for calculating the strains.

(i) When X_u /d lies between 0.001 and 0.1 (neutral axis is considered to be above compression steel), then the compression steel will be in tension. Hence, as per similar triangles, the formula for finding the strain in compression steel is obtained as given below:

$$\varepsilon_{sc} = \frac{0.0035[\left(\frac{d'}{d}\right) - \left(\frac{X_u}{d}\right)]}{X_u/d}$$

(ii) When X_u /d is greater than 0.1(i.e., neutral axis is below the compression steel), then the compression steel will always be in compression and applying the strain compatibility (similar triangles), the formula for finding strain in compression steel is

$$\varepsilon_{sc}=0.0035[1-d'/X_u]$$

Using the above algorithm, a MATLAB code is generated to calculate the ultimate moment of resistance without considering the partial safety factors for materials for any beam, given all its input. The generated code is provided in the appendix for further review.

A) Uncertainties in Construction

In analysis and design calculations, the major variables subjected to high degree of uncertainty considered in our study are:

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1. Loads
2. Material Properties
3. Dimensions

There are many other unforeseen variable factors which affects the strength and serviceability of structures which we have not taken into account in our investigation. For practical and quantitative representation of reliability, statistical and probabilistic analysis is quite rational. Recent attempts include the application of fuzzy logics also.

B) Deterministic and Probabilistic Approaches

In the presence of uncertainty, it is not simple to satisfy the basic design requirements. The design depends upon the following two parameters:

1. Load on the structure 'S' (lifetime maximum)
2. Ultimate resistance 'R' (capacity of structure)

Both S & R are expressed in terms of stress resultant such as bending moment at critical section, and are treated as statistically independent random variables; their randomness is characterized by their means (μ_S, μ_R), standard deviations (σ_S, σ_R) and corresponding probability density functions.

If **S<R**, the structure is expected to be safe, and if **S>R**, the structure is expected to fail. The probability of failure can be calculated as follows:

P_f**= Probability [{R < S} ^ {0<S<∞}]**

Reliability Analysis:

A rational and quantitative solution to the problem of 'adequate safety' can be obtained by quantifying the acceptable risk in terms of target probability of failure or target reliability.

Reliability is expressed as **the complement of the probability of failure**, i.e.

Reliability = (1- P_f**)**

Evaluating the probability of failure (P_f) (or the reliability) underlying a given structure is termed as **reliability analysis**, whereas designing a structure to meet the target reliability is termed as **reliability design**.

Reliability analysis of moment capacity of RC beam:

Theoretically we do not consider the deviation of M_{ur} of a given RC beam, but practical cases include many uncertainties. Thus ultimate moment of resistance for the beam can never be a fixed (calculated) value. The possible cases of uncertainties which have been taken into consideration are:

1.) Dimensions of beam (b, d)
2.) Grade of concrete (f_{ck})
3.) Grade of steel (f_y)
4.) Percentage of tension steel provided (P_t)
5.) Percentage of compression steel provided (P_c)
6.) Load acting on the beam (Q)

All these factors vary accordingly from their nominal values over a range of deviation, and their respective type of probability distribution patterns. As per results of statistical analysis of variation of these parameters, mean, coefficient of variation (C.O.V) and probability distributions are obtained by various established studies and a series of random variables can be generated which represent the most probable values of these parameters with in which they lie. Table 4 gives the results of statistical analysis of variation of parameters taken into consideration for the generation of random variables.

TABLE 4

RESULTS OF STATISTICAL ANALYSIS TAKEN INTO CONSIDERATION

Variable	Mean	C.O.V	Probability distribution
Compressive strength of concrete (Nominal mix- M20)	19.54	0.21	Normal
Yield Strength of steel	Fe 250 – 320 Fe 415 – 469	0.1	Normal
Dead load (D)	1.05D	0.1	Normal
Flexural reinforcement	Mean of Provided/ required= 1.01	0.04	Lognormal

TABLE 5

RESULTS OF STATISTICAL ANALYSIS OF VARIATION IN DIMENSIONS

Beam	Mean Deviation	Standard Deviation	Size range(mm)
Breadth	+10.29	9.47	200-350
Effective depth	+6.25	3.79	250-370
Overall depth	+14.37	9.38	250-700

For this range of parameters a certain range of ultimate moment of resistance of a beam is calculated. This can be checked with external moment on the beam due to load (self - weight) acting on the beam and the possible number of failure cases can be verified. Thus reliability, which is nothing but the complement of probability of failure, is obtained.

In the present research work, we have assumed a set of inputs such as **b=275mm, d=320mm,** f_{ck} **=20 MPa,** f_y**=415 MPa** for which ten thousand random variables are generated and the corresponding values obtained in Excel files are called in MATLAB code by '**dlmread**' command. For any given P_c, P_t and span of the beam the reliability analysis of beam is calculated. If $M_{ur} < M_e$ the beam is expected to fail. Thus counting the probable cases of failure we can evaluate the reliability of the reinforced beam.

IV. RESULTS

Using the MATLAB generated code, the ultimate moment of resistance of any doubly reinforced beam can be calculated for the given percentage of compression and tension steel and span of the beam. The code written is flexible for any given reinforced beam, the 'dlmread' command takes the set of random variables generated by various mathematical simulations for the given

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corresponding input data. The quantification of number of cases of probabilistic failure and the complementary reliability analysis is done. This verifies the reliability of the beam.

As mentioned earlier, we have assumed a beam of width b=275mm, effective depth d=320, grade of concrete fck =20 MPa and grade of steel fy =415 and generated a set of 10,000 random variables by using MATLAB inbuilt command and considering the distribution and mean and co-efficient of variations as mentioned above in the Tables 4 & 5 and tested the code for various cases of Pc and Pt and performed the reliability analysis of the beam with the given input data and, we have mentioned two such results. Likewise the probabilistic study can be carried for various other combinations of input data and reliability of the beam can be verified. Two examples have been cited for each of the cases, when the beam is singly reinforced and doubly reinforced.

Test result 1:

Input data: Pc =0.5, Pt =1.0, span of beam=6000mm; d'=40mm;

Probability of failure (Pf) = 0; Reliability=1

Hence for this combination of given input the beam is very reliable.

Test result 2:

Input data: Pc =0.0, Pt=0.9, span of beam=6000mm; d'=50mm;

Probability of failure (Pf) = 0.0263; Reliability=0.9737

For the given input data the beam is reliable for 97 beams out of 100, hence this combination can be discarded.

V. Conclusions

According to the study, the probability of failure is computed for the various possible cases of input, where in 10,000 possible values for each of the random variables are considered including uncertainties and neglecting partial safety factors for the materials. The computation of the Ultimate Moment resistant capacity of the beams is in order to obtain the true strength and failure possibility of a beam. The results can be concluded as:

1) For any given doubly reinforced beam the probability of failure has been obtained as zero.

2) Whereas for a singly reinforced beam the probability of failure depends upon the percentage of steel.

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