

MHD Rivlin-Ericksen Free Convective Memory Flow through Porous Medium

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Abstract – Effects of free convective two dimensional unsteady Rivlin-Ericksen memory flow through porous medium of variable permeability, bounded by vertical porous plate with uniform suction and constant heat flux under the influence of uniform transverse magnetic field are studied. The permeability of porous medium fluctuates with time about a constant mean. Approximate solutions are found for mean velocity, transient velocity and temperature distribution. The temperature distribution and mean velocity are shown graphically and discussed.

Index Terms – Magneto hydrodynamics, Free convection, Rivlin-Ericksen memory fluid, Porous medium.

I. INTRODUCTION

In recent years, the problem of free convection flow through a porous medium of variable permeability has attracted the attention of many scholars. Sreekanth et al. [3] studied the fluctuating free convection through porous medium due to an infinite vertical plate with variable permeability. Rees and Pop [5] investigated free convective flow in porous medium of variable permeability effects. Singh et al. [6] have studied MHD free convective viscous flow past the vertical porous plate through non-homogenous porous media with radiation and temperature gradient dependent heat source in slip flow regime. Batarseh and Duwawri [7] have studied Isentropic sound propagation analysis and optimization over flat plate of saturated porous media of variable permeability. Noushima et al. [8] have studied Hydro Magnetic free convective Rivlin – Erickson flow through a porous medium of variable permeability. Uwanta et al. [9] analysed the heat and mass transfer flow through porous medium of variable permeability. Babu and Satya[10] have studied the influence of variable permeability and radiation absorption on heat and mass transfer in MHD micro-polar flow over a vertical moving porous plate. The variable permeability problem bounded by vertical plate has possible application in design of steam displacement process in oil recovery and geothermal systems.

Aim of the author is to extend the problem of [4] to Rivlin-Ericksen fluid [1] under the influence of transverse magnetic field.

II. FUNDAMENTAL EQUATIONS

The convective memory flow through a porous medium bounded by an infinite vertical porous plate with constant heat flux subject to uniform transverse magnetic field is considered. x- axis is taken along the plate in the upward direction and y-axis normal to it .

All the fluid properties are assumed to be constant, except the influence of density variations with temperature is considered only in the body force term. The magnetic field of small intensity H_0 is induced in the 'y' direction since the fluid is slightly conducting, the magnetic Reynolds number is very much less than unity hence the induced magnetic field is omitted in comparison with the applied magnetic field. The viscous dissipation and Darcy's dissipation terms are omitted for small velocities. The flow in the medium is entirely due to buoyancy force. Under these conditions, the governing equations of flow are

$$\frac{\partial v}{\partial y} = 0 \quad \text{i.e } v = -v_0 \quad (2.1)$$

$$\begin{aligned} \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = g\beta_1(T - T_\infty) + g \frac{\partial^2 u}{\partial y^2} \\ + \frac{\beta}{\rho} \left[\frac{\partial^2 u}{\partial t \partial y^2} + v \frac{\partial^2 u}{\partial y^3} \right] - \frac{\partial u}{k(\tau)} \\ - \left(\frac{\sigma \mu_0^2 H_0^2}{\rho} \right) u \end{aligned} \quad (2.2)$$

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2} \quad (2.3)$$

Where u and v are velocity components along x and y axes, and ' k ' is the permeability of porous medium.

$$k(\tau) = k_0(1 + \epsilon e^{i\omega\tau}) \quad (2.4)$$

Where ' k_0 ' is the mean permeability of the medium, ' ω ' the frequency of fluctuation, ' τ ' the time and ' ϵ ' ($\ll 1$) is a constant quantity.

The boundary conditions are:

$$y = 0, \quad u = 0, \quad \frac{\partial T}{\partial y} = -\frac{q}{\kappa}$$

$$y \rightarrow \infty, \quad u = 0, \quad T = T_\infty \quad (2.5)$$

Introducing the following non-dimensional quantities:

$$y' = \frac{y v_0}{\delta}, \quad t' = \frac{t v_0^2}{4\delta^2}, \quad \omega' = \frac{4\delta \omega}{v_0^2}$$

$$M = \frac{\sigma \delta H_0^2 \mu_e^2}{\rho v_0^2}, \quad u' = \frac{u}{v_0}, \quad P = \frac{\mu C_p}{\kappa}$$

$$G = \frac{g \beta_1 q \delta^2}{\kappa v_0^4}, \quad k'_0 = \frac{k_0 v_0^2}{\delta^2}, \quad R = \frac{\beta_1 v_0^2}{\delta^2}$$

$$\theta = \frac{(T - T_\infty) \kappa v_0}{q \delta} \quad (2.6)$$

The equations (2.2) and (2.3), in view of (2.4) and (2.6) reduce to the following form.

$$\frac{1}{4} \frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = G\theta + \frac{\partial^2 u}{\partial y^2} + R \left(\frac{\partial^3 u}{4 \partial t \partial y^2} - \frac{\partial^3 u}{\partial y^3} \right) - \frac{u}{k_0(1 + \varepsilon e^{i\omega t})} - Mu \quad (2.7)$$

$$\frac{1}{4} \frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{P} \left(\frac{\partial^2 \theta}{\partial y^2} \right) \quad (2.8)$$

The corresponding boundary conditions become

$$y = 0, \quad u = 0, \quad \frac{\partial \theta}{\partial y} = -1$$

$$y \rightarrow \infty, \quad u = 0, \quad \theta = 0 \quad (2.9)$$

III. SOLUTION OF THE PROBLEM

The partial differential equations (2.7) and (2.8) are reduced to ordinary ones by assuming the following series expressions for velocity and temperature fields.

$$u(y, t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y) \quad (3.1)$$

$$\theta(y, t) = \theta_0(y) + \varepsilon e^{i\omega t} \theta_1(y) \quad (3.2)$$

Substituting equations (3.1) and (3.2) in equations (2.7) and (2.8) and equating the coefficients of like powers of ε to zero, the differential equations so obtained are

$$Ru_0''' - u_0'' - u_0' + \left(\frac{1}{k_0} + M \right) u_0 = G\theta_0 \quad (3.3)$$

$$Ru_1''' - \left(1 + \frac{Ri\omega}{4} \right) u_1'' - u_1' + \left(\frac{1}{k_0} + M + \frac{i\omega}{4} \right) u_1$$

$$= G\theta_1 + \frac{u_0}{k_0} \quad (3.4)$$

$$\left(\frac{1}{P} \right) \theta_0'' + \theta_0' = 0 \quad (3.5)$$

$$\left(\frac{1}{P} \right) \theta_1'' + \theta_1' - \left(\frac{i\omega}{4} \right) \theta_1 = 0 \quad (3.6)$$

The corresponding boundary conditions become

$$\theta_0' = -1, \theta_1' = u_0 = u_1 = 0 \text{ as } y = 0$$

$$\theta_0 = 0, \theta_1 = 0 = u_0 = u_1 = 0 \text{ as } y \rightarrow \infty \quad (3.7)$$

The differential equations (3.3) and (3.4) are of third order when $R \neq 0$ and two boundary conditions are there. So Beard and Walters [2] rule gives rise to,

$$u_0 = u_{01} + Ru_{02} + o(R^2) \quad (3.8)$$

$$u_1 = u_{11} + Ru_{12} + o(R^2) \quad (3.9)$$

Substituting these equations (3.8) and (3.9) into (3.3) and (3.4) equating different powers of R and neglecting, $o(R^2)$, there by

$$u_{01}'' + u_{01}' - \left(\frac{1}{k_0} + M \right) u_{01} = -G\theta_0 \quad (3.10)$$

$$u_{11}'' + u_{11}' - \left(\frac{1}{k_0} + M + \frac{i\omega}{4} \right) u_{11} = G\theta_1 - \frac{u_{01}}{k_0} \quad (3.11)$$

$$u_{02}'' + u_{02}' - \left(\frac{1}{k_0} + M \right) u_{02} = u_{01}''' \quad (3.12)$$

$$u_{12}'' + u_{12}' + \left(\frac{1}{k_0} + M \right) u_{12} = u_{11}''' - \left(\frac{i\omega}{4} \right) u_{11}'' - \frac{u_{02}}{k_0} \quad (3.13)$$

The boundary conditions are

$$u_{01} = u_{02} = u_{11} = u_{12} = 0 \text{ as } y \rightarrow 0$$

$$u_{01} = u_{02} = u_{11} = u_{12} = 0 \text{ as } y \rightarrow \infty \quad (3.14)$$

The velocity and temperature fields are given by

$$u = u_0 + \varepsilon u_1$$

$$u = u_{01} + Ru_{02} + \varepsilon (u_{11} + Ru_{12}) = u_{01} + Ru_{02} + \varepsilon u_{11} \quad (3.15)$$

$$\theta = \theta_0 + \varepsilon \theta_1 \quad (3.16)$$

Solving (3.10 - 3.13) under boundary conditions (3.14), taking real part of the solution and can be put into form of fluctuating parts

$$u(y, t) = u_0(y) + \epsilon[M_1 \cos \omega t - M_2 \sin \omega t] \quad (3.17)$$

The expression of transient velocity profiles for $\omega t = \pi/2$ is given by

$$u(y, \pi/2\omega) = u_0(y) - \epsilon M_2 \quad (3.18)$$

Where

$$u_0(y) = \frac{G}{P_r} [(e^{-Pr y} - e^{-Pr}) (1 - R P_r) - R r_2 e^{-r_2 y}] \quad (3.19)$$

$$\theta(y, t) = \left(\frac{1}{Pr}\right) e^{-Pr y} \quad (3.20)$$

For the sake of brevity constants and fluctuating parts are not given.

CONCLUSIONS

The temperature distribution decreases as Prandtl number increases, is shown in fig 1. The mean velocity profiles are plotted in fig 2. Mean velocity increases with Grashoff number 'Gr' and Permeability parameter 'k₀' but decreases with Prandtl number P_r and Hartmann number 'M'.

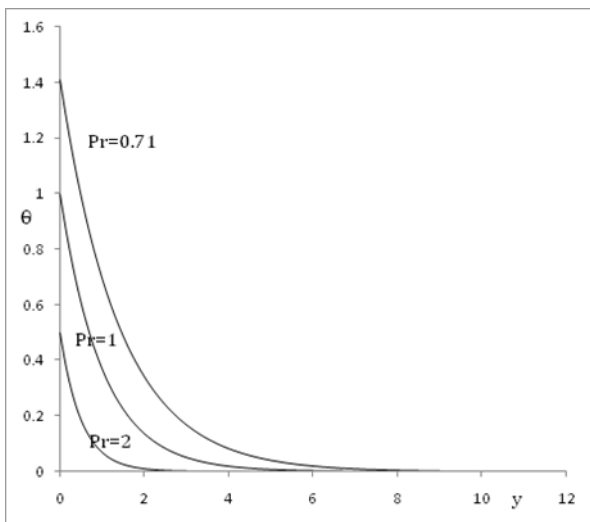


Figure 1. The temperature profiles for different values of Prandtl number

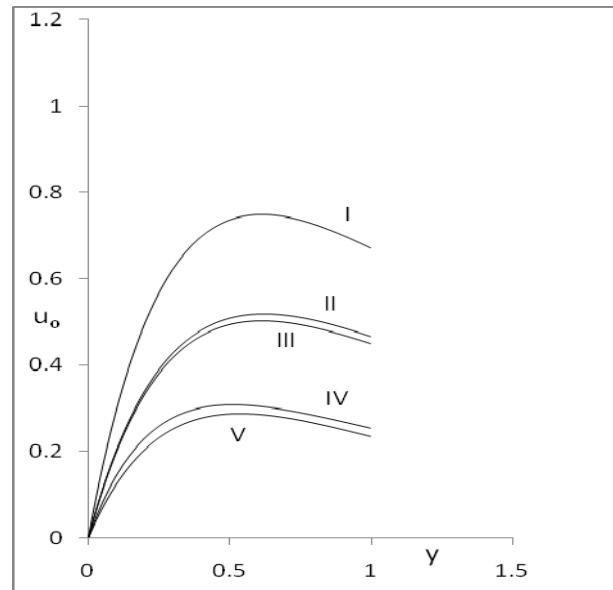


Figure . 2. Effects of k₀, M, Gr, P_r on mean velocity u₀ versus y for R = 0.05

	P _r	M	Gr	k ₀
I	0.71	5	6	3
II	0.71	5	4	10
III	0.71	5	4	3
IV	0.71	10	4	3
V	1	5	4	3

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