Numerical Solution Of Oscillatory Motion Of Dusty Memory Fluid Through Porous Media

D.Sreenivas[1] and B.Shankar[2]
[1]CVR College of Engineering, Department of H&S, Ibrahimpatan, R.R.District, A.P., India
Email:srinivasdhonthi@gmail.com
[2]Osmania University, Department of Mathematics, Hyderabad, A.P., India
Email: headmath1ou@yahoo.co.in

***Abstract*—The solution of oscillatory motion of dusty memory fluid through porous media by finite element method is obtained; expressions for the velocity of fluid, dust and skin friction are also obtained. The effects of various parameters on above are shown graphically and discussed.**

***Index Terms*—Oscillatory motion, Dusty memory fluid, Porous medium.**

I. Introduction

The study of flow dusty fluids has important applications in the fields of fluidization, combustion use of dust in gas cooling systems, centrifugal separation of matter from fluid, petroleum industry, purification of crude oil, polymer technology and fluid droplets sprays.

The constitutive equation for the rheological equations of state for a memory fluid (Walter's liquid B model) given by Walter (1960, 1962).Grover (1968) studied the motion of an incompressible viscous fluid bounded by two infinite plates, the upper one is fixed and the other executing a simple harmonic oscillation in its own plane. Siddappa and Shanker Hegde (1972) have extended the Grover's work for oscillatory motion of memory fluid given by Rivlin-Ericksen constitutive equation. Sattar and Ahmed (2007) obtained the numerical solution of Non-Newtonian fluid. Rafiuddin et al. (2006) derived the exact solution of oscillatory motion of a memory fluid through porous media with a horizontal force. Ramu et al. (2010) presented the numerical solution of the above problem.

The aim of the present investigation is to study oscillatory motion of dusty memory fluid through porous medium which is bounded by two infinite parallel plates and both the plates are vibrating with same amplitude and frequency.

II. Formulation Of The Problem

Consider the oscillatory motion of a dusty memory fluid bounded by two infinite parallel plates through a porous medium. Let the direction of motion of the fluid be along the x-axis, which is chosen along the lower plate and the y-axis be perpendicular to it. Let (u, v, w) be the velocity components of the fluid. For the present study, v = w = 0**.** The velocity of the oscillating plate at any instant is taken as $u = \mathrm{Re}\,(u_0 e^{-i\omega t})$, where 'Re' represents the real part. For convenience we drop the expression 'Re', but we take the real part of the final result. The equation of motion governing the dusty memory flow is of the form

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial y^2} + \frac{KN_o}{\rho}(v_p - u) - \beta \frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{\alpha u}{\gamma} \tag{2.1}$$

$$\frac{\partial v_p}{\partial t} = \frac{K}{m}(u - v_p) \tag{2.2}$$

and the equation of continuity is

$$\partial u/\partial x = 0 \tag{2.3}$$

The equation will now be made dimensionless by introducing the non-dimensional quantities.

$$y' = \frac{y}{y_o}, \quad u' = \frac{u y_o}{\alpha}, \quad t' = \frac{\alpha t}{{y_o}^2}$$
$$\tau = \frac{m\alpha}{K{y_o}^2}, \quad \gamma' = \frac{\gamma}{{y_o}^2}, \quad v_p' = \frac{y_o v_p}{\alpha}, \quad s = \frac{\beta}{y_0^2} \tag{2.4}$$

Substituting in (2.1) & (2.2) dropping the dashes for simplicity, we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \left(\frac{l}{\tau} + \frac{1}{\gamma}\right) u - s\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right) + \frac{l v_p}{\tau} \tag{2.5}$$

$$\tau \frac{\partial v_p}{\partial t} = (u - v_p) \tag{2.6}$$

Eliminating v_p from (2.5) making use of (2.6), we get

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial t}\right) + \left(\frac{1}{\tau}\right)\left(\frac{\partial^2 u}{\partial y^2} - s\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right)\right) - \frac{\partial u}{\partial t}\left(\frac{l+1}{\tau} + \frac{1}{\gamma}\right) - s\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{u}{\tau\gamma} \tag{2.7}$$

Where

K– stock's coefficient of resistance (6πaμ) for spherical dust particles,
a – average radius of dust particle,
μ – viscosity of the fluid,
l– mN_o/ρ mass concentration of dust particle,

DOI:10.32377/cvrjst0214

ρ – density of fluid,
m –average mass of dust particle,
N_o– number of dust particle per unit volume,
β–kinematic visco-elasticity
τ – m/K relaxation time,
α – kinematic viscosity,
γ –permeability parameter,
y_o –characteristic velocity,
s – memory parameter.

III. Solution Of The Problem

Let the lower plate execute simple harmonic oscillations in its own plane whereas the upper plate is fixed .In this case the boundary conditions are

$$y = 0, \quad u = u_o e^{-i\omega t},$$
$$y = 2y_o, \quad u = u_o e^{-i\omega t} \qquad (3.1)$$

Where $2y_0$ is the clearance distance between the vibrating plate and fixed plate.

Introducing dimensionless frequency ω′given by $\omega' = \omega y_o^2/\alpha$ and using (2.4), the boundary conditions in (3.1) in dimensionless form reduces to

$$y = 0, \quad u = e^{-i\omega t},$$
$$y = 2, \quad u = e^{-i\omega t} \qquad (3.2)$$

To solve the equation (2.5), we assume the solution of the form

$$u = g(y)\, e^{-i\omega t} \qquad (3.3)$$

Now applying the boundary conditions (3.2) to (3.3), we get

$$y = 0, \quad g(y) = 1,$$
$$y = 2, \quad g(y) = 1 \qquad (3.4)$$

Substituting (3.3) into (2.5), we have

$$g''(y) + m g(y) = 0 \qquad (3.5)$$

Where

$$m = \frac{(-1+\omega^2\tau\gamma)+i\omega[(l+1)\gamma+\tau]}{\gamma[(1+\omega^2\tau s)-i\omega(\tau-s)]} \qquad (3.6)$$

The equation (3.5) is an ordinary differential equation with boundary conditions (3.2) through finite element method using Galerkin method, the solution of (3.5) is given by

$$g(y)=1+ay(y-2) \qquad (3.7)$$

The velocity distribution is given by (3.3)

The real part of the velocity of the fluid is given by

$$u(y)=[1+G_1 y(y-2)]\cos\omega t+[H_1 y(y-2)]\sin\omega t \qquad (3.8)$$

Skin friction at the lower plate is given by

$$C=\left(\frac{\partial u}{\partial y}\right)_{y=0} = -2(G_1\cos\omega t + H_1\sin\omega t) \qquad (3.9)$$

Dust velocity is given by

$$V_p=\frac{\tau}{l}\left[\begin{array}{l}\sin\omega t(-\omega-\omega G_1 y(y-2)-2H_1-2s\omega G_1+(\frac{l}{\tau}+\frac{1}{\gamma})H_1 y(y-2))+\\ \cos\omega t(H_1 y(y-2)\omega-2G_1+2H_1\omega s+(\frac{l}{\tau}+\frac{1}{\gamma})(1+G_1 y(y-2))\end{array}\right] \qquad (3.10)$$

$$D=\left(\frac{\partial V_p}{\partial y}\right)=\frac{\tau}{l}\left[\sin\omega t(2\omega G_1-(\frac{l}{\tau}+\frac{1}{\gamma})2H_1)+\cos\omega t(-2H_1\omega-2(\frac{l}{\tau}+\frac{1}{\gamma})G_1)\right] \qquad (3.11)$$

Where the constants are not given for the sake of brevity

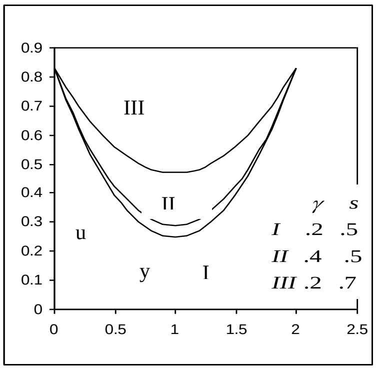

Fig 1.velocity profile for fixed values of $\omega=1, t=0.6, l=0.2, \tau=2$

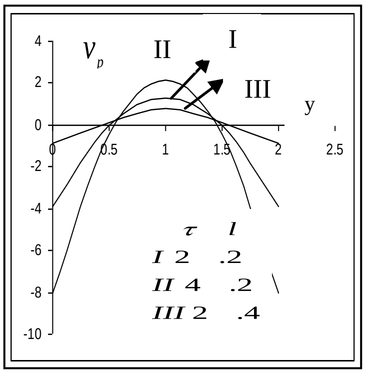

Fig 2.dust velocity profile for fixed values of $\omega=1, t=0.6, s=0.5, \gamma=2$

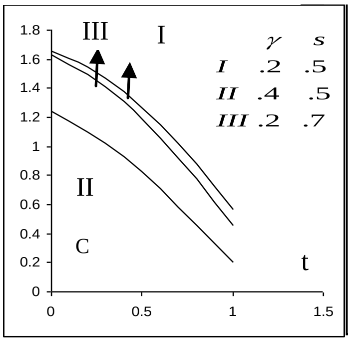

Fig 3.skin friction for fixed values of $\omega=1, t=0.6, l=0.2, \tau=2$

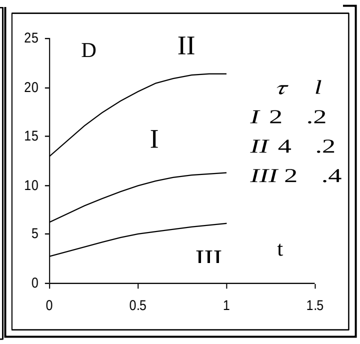

Fig 4.skin friction of dust profile for fixed values of $\omega=1, t=0.6, s=0.5, \gamma=2$

Discussion And Conclusion

From fig-1 it is found that as permeability parameter (γ) and memory parameter (s) increase, fluid velocity (u) increases and from fig-2 we see that mass concentration (*l*) increases dust velocity (V_p) decreases and V_p increases with relaxation time (τ) , from the middle of the plates the trend is opposite for both. From fig-3 it is observed that skin friction (C) decreases with permeability parameter and memory parameter. Skin friction in fig-4 for the dust case (D) decreases with mass concentration and increases with relaxation time.

We can further extend this work by studying oscillatory motion of dusty memory fluid through porous medium which is bounded by two infinite parallel plates by applying horizontal force following Lamb.

References

[1] Walter, K. Quart. Jour. Mech. Applied Math. 13, pp. 444, 1960.
[2] Walter, K. Quat. Jour. Mech Applied Math. 15 pp. 136, 1962.
[3] Grover, G.K Proc. Int. Soc. Theoretical and Applied Mechanics pp.89, 1968.

[4] Siddappa, B. and Shanker Hegde Progress of Mathematics B.H.U., 6 (2):pp15, 1972.
[5] Rajput (1991) M.Phil. Thesis, Gulberga University, Gulberga.
[6] Arjumand, S. and Ahmed Waheedullah Journal of pure and applied physics vol. 19 No.2 (April- June) pp. 69-75, 2007.
[7] Addepalli Ramu, M.V. Ramana Murthy, Rafiuddin, Arjumand Sattar and Adeeb Nazia Atti Della Fondazione Giorgio Ronchi sept-oct; LXV 5, pp. 607-618, 2010.
[8] Lamb, S.Horance 1932:Hydrodynamics.Sixth edition Cambridge University Press.
[9] Syed Amanullah Hussaini , M V Ramana Murthy, Ahmed Waheedullah, Rafiuddin-Numerical solution of oscillatatory motion of a dusty viscous fluid-Atti Della "Fondazoine Giorgio Ronchi"-Anno LXVI,2011-N.6.