**B.Tech**. I Year II Sem. *2017- 18*
**Model Paper-1 (MID-I)**
Subject**: Mathematics-II (Common to All)**

## Short Answer Questions

1. Find the value of '$k$' such that the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is two.
2. If 2, 3 are eigenvalues of $A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 0 \\ a & 0 & 2 \end{bmatrix}$ then find $a$.
3. Find the matrix of the quadratic form $-3x_1^2 - 3x_2^2 - 3x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3$
4. Is the matrix $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ diagonalizable?
5. Show that $A=\begin{bmatrix} a+ic & -b+id \\ b+id & a-ic \end{bmatrix}$ is unitary matrix if $a^2+b^2+c^2+d^2=1$

## Long Answer Questions

6. a) Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ by reducing to normal form.

b) Determine whether the following system is consistent and find its solution.
$x-2y+z+2w=1$, $x+y-z+w=2$, $x+7y-5z-w=4$.

**[OR]**

7. a) Determine the values of '$k$' for which the system of equations
$x-ky+z=0$
$x+3y-kz=0$ has (i) Only trivial solution (ii) Non- trivial solution
$3x+y-z=0$

b) Prove that the Eigen values of a Hermitian matrix are real.

8. Determine the modal matrix $P$ of $A=\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. Verify that $P^{-1}AP$ is a diagonal matrix.

**[OR]**

9. Reduce the quadratic form $3x_1^2+3x_2^2+3x_3^2+2x_1x_2+2x_1x_3-2x_2x_3$ into canonical form by orthogonal transformation. Hence find its rank, index, signature and nature.

10. a) Verify Cayley-Hamilton theorem for $A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$. Hence find $A^{-1}$?

b) The eigen vectors of a $3\times3$ matrix A corresponding to the Eigen values 1, 1, 3 are $\begin{pmatrix}1\\0\\-1\end{pmatrix}, \begin{pmatrix}0\\1\\-1\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}$. Find the matrix A?

**[OR]**

11. a) Reduce the quadratic form $6x_1^2+3x_2^2+3x_3^2-4x_1x_2+4x_1x_3-2x_2x_3$ into canonical form by elementary transformation (congruence reduction) and discuss its nature

b) Reduce the quadratic form $x^2+6y^2+18z^2+4xy+8xz-4yz$ into canonical form by Lagrange's reduction and discuss it's nature.

# Model Paper-2 (MID-I)

**PART – A**

Answer **ALL** questions **5x2=10 Marks**

1. Examine the linear independence of the set of vectors over **R** $\{(2,2,0,2),(4,1,4,1),(3,0,4,0)\}$
2. *I*f a 3x3 matrix *A* has eigen values 1, 2, -1. Find the trace of the matrix $B=A-A^{-1}+A^2$.
3. Is the matrix A=$\begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}$ diagonalizable?
4. Show that the linear transformation $\begin{pmatrix}x_1\\x_2\end{pmatrix}$ to $\begin{pmatrix}3x_1+2x_2\\4x_1+x_2\end{pmatrix}$ is a non singular transformation.
5. Show that the matrix $A=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1+i\\1-i & -1\end{bmatrix}$ is unitary.

**PART – B**

Answer **ALL** questions **3x10=30 Marks**

6. a) Reduce the matrix into echelon form and find it's rank: $\begin{bmatrix}2 & 0 & -1 & 0\\4 & 1 & 0 & 5\\0 & 1 & 3 & 6\\6 & 1 & -2 & 6\end{bmatrix}$

b) Solve $\begin{aligned}3x+4y-z-6w&=0\\2x+3y+2z-3w&=0\\2x+y-14z-9w&=0\\x+3y+13z+3w&=0\end{aligned}$

**OR**

7. a) Find the rank of the matrix by reducing to normal form: $\begin{bmatrix}2 & 3 & 1 & 0 & 4\\3 & 1 & 2 & -1 & 1\\4 & -1 & 3 & -2 & -2\\5 & 4 & 3 & -1 & 5\end{bmatrix}$

b) Investigate the values of $\lambda$ and $\mu$ so that the equations

$$2x+3y+5z=9, 7x+3y-2z=8, 2x+3y+\lambda z=\mu$$

have i) no solution ii) a unique solution iii) an infinite solutions.

8) Diagonalize the matrix A= $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$ , and hence find $A^4$.

**OR**

9) a) Find the matrix A whose eigen values and eigen vectors respectively are1, -1, 2 and

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$$

b) Reduce the Quadratic form $2x^2+7y^2+5z^2-8xy+4xz-10yz$ to canonical form using lagrange's reduction.Hence find the nature, rank, index and signature.

10) a) Verify cayley Hamilton theorem for $A=\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence find $A^{-1}$.

b) Show that the eigen values of a skew-hermitian matrix are purely imaginary or zero.

**OR**

11) a) Reduce the Quadratic form to canonical form using orthogonal reduction $x_1^2+2x_2^2-7x_3^2-4x_1x_2+8x_1x_3$ and write its orthogonal transformation.

## ASSIGNMENT-1

### Short answer Questions:

1. If $A$ and $B$ are symmetric matrices then prove that $AB$-$BA$ is skew-symmetric.
2. Find the value of '$k$' such that the rank of the matrix $\begin{bmatrix} 1 & -2 & 1 \\ 2 & k & -2 \\ 3 & 1 & -1 \end{bmatrix}$ is 2.
3. Examine the linear independence of the set of vectors over $R^3$ $\{(1,3,4),(1,1,0),(1,4,2),(1,-2,1)\}$
4. If $A$=$\begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix}$ then find the eigen values of $A^3-A^2-I$
5. If $A$=$\begin{bmatrix} 2 & 3 & 4 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ then find the eigen values of adj $A$
6. Let a $4\times 4$ matrix $A$ have eigen values 1, -1, 2, -2. Find the value of the determinant of the matrix $B=2A+A^{-1}-I$
7. Show that the transformation $y_1=x_1\cos\theta+x_2\sin\theta$, $y_2=-x_1\sin\theta+x_2\cos\theta$ is orthogonal.

**Long answer Questions:**

1. Reduce the matrix into echelon form and find it's rank: $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}$

2. Find the rank of the matrix by reducing to normal form: $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & -4 \\ 2 & 3 & 5 & -5 \\ 3 & -4 & -5 & 8 \end{bmatrix}$

3. Find the matrices P and Q such that PAQ is in the normal form, where A= $\begin{bmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{bmatrix}$

4. Solve the system of equations: $\begin{aligned} x-2y+z+2w&=1 \\ x+y-z+w&=2 \\ x+7y-5z-w&=4 \end{aligned}$

5. Solve $\begin{aligned} 3x+4y-z-6w&=0 \\ 2x+3y+2z-3w&=0 \\ 2x+y-14z-9w&=0 \\ x+3y+13z+3w&=0 \end{aligned}$

6. Verify cayley Hamilton theorem for A= $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & -1 \\ -2 & -1 & 1 \end{bmatrix}$ and hence find $A^{-1}$

7. Find the eigen values and eigen vectors of A= $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

8. Prove that eigen values of a Hermitian matrix are real.

9. Diagonalise the matrix $\begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

10. Is the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ diagonalizable?

11. Find the matrix A whose eigen values and eigen vectors are 1, 2, 3 and $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix}$

12. Reduce the Quadratic form to canonical form using orthogonal reduction $2xy+2yz+2zx$. Find nature, index and signature.

13. Reduce the Quadratic form to canonical form using lagrange's reduction $x_1^2+7x_2^2+7x_3^2+4x_1x_2-18x_2x_3-6x_3x_1$

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